Validation of Water Waves Reproduced by Smoothed Particle Hydrodynamics

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1. INTRODUCTION

Wave generation and propagation is an important issue in naval architecture and ocean engineering, so many theoretical, experimental and numerical studies has been done. Treatment of nonlinear waves is one of major problems in research fields of seakeeping, stability and maneuvering of ships, e.g. green water, slamming, propeller racing, capsizing, and ship handling in adverse conditions. Recently strongly nonlinear waves such as breaking waves, freak waves and tsunamis are highlighted because they could lead to serious accidents/disasters. Application of CFD is straightforward way nowadays to deal with such nonlinear waves thanks to the rapid advancement of computer performance. CFD can handle any events/situations where nonlinear waves play an important role, so it is possible to remove limitations of experiment if computer resources are sufficient enough.

In the present paper we try to reproduce wave generation and propagation in shallow waters using SPH (Smoothed Particle Hydrodynamics). Although SPH has been applied to many nonlinear wave problems[1-2], there are few validation studies on wave generation and propagation by directly imposing movement of wave maker. Therefore we need at first to confirm the validity of water waves reproduced by SPH before tackling realistic engineering problems previously mentioned. We use DualSPHysics for this study, which is an open source GPU-accelerated SPH solver.

A validation experiment was conducted at a shallow water wave basin of Kobe University. Wave generation experiment was done for three different water depths, covering deep to shallow waters, using a piston-type wave maker with three amplitudes and two periods of piston motion. In SPH simulation, wave generation and propagation is reproduced by imposing the motion of piston board recorded in the experiment. As a first step, validation of SPH method for long mild waves in shallow waters where seabed effects cannot be neglected is attempted. As a result, the SPH simulation can well reproduce fundamental features of shallow-water waves in terms of wave amplitude, wave profile and phase velocity for all water depths except for wave pressure.

2. EXPERIMENT

2.1 Experimental setup

An experiment was done at a wave basin of Kobe University (Fig.1), whose length is 60 m, width is 6 m and water depth is 0 to 1.5 m, and a piston-type wave maker is equipped.

In this experiment we try to capture wave generation and propagation using a wave gauge (WG) which is placed at 24.6 m distance from the neutral position of piston. Pressure sensor is also placed to measure wave pressure at the bottom of wave basin at the same distance with the wave gauge as shown in Fig.2. Piston motion was directly measured using a laser distance sensor that was attached to the non-moving frame in front of the piston board as shown in Fig.3. Therefore we can capture a piston motion and use it directly in numerical simulation to generate waves.

In addition, wave profile is also recorded by a video camera at the same position of wave gauge and pressure sensor through an observation window. By use of these experimental data, i.e. time history of piston motion including gradual start at beginning, time history of wave elevation measured at a position sufficiently far from the wave maker, wave profile given as video image and pressure variation at the bottom of basin due to wave passage.

Fig.1 Shallow water wave basin

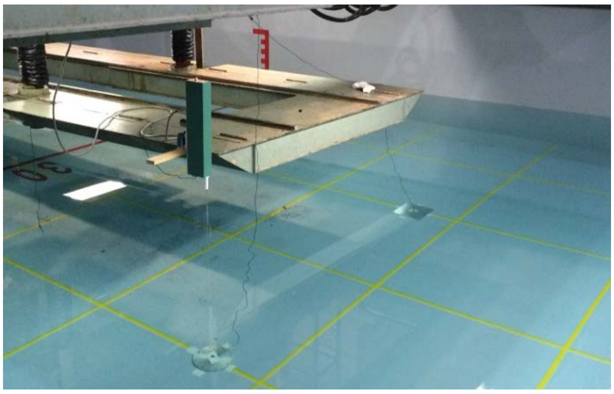

Fig. 2. Wave gauge and pressure sensor

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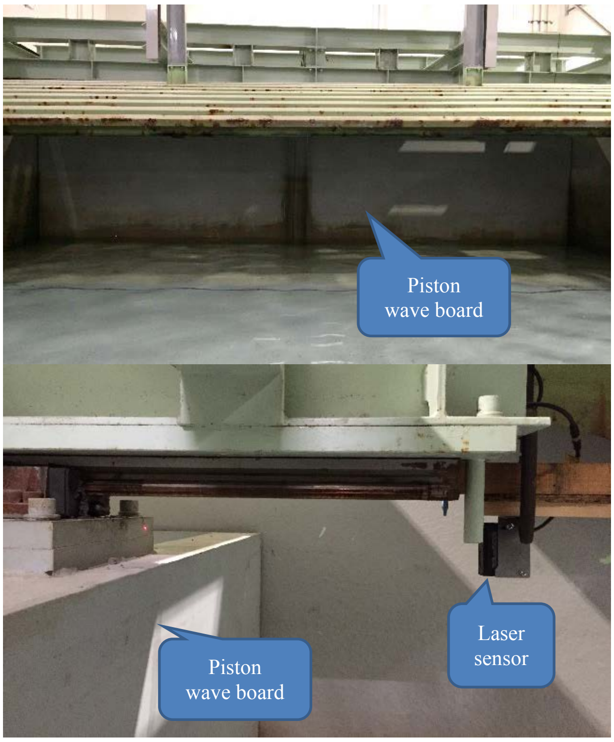

Fig. 3. Measurement of piston motion

2.2 Wave condition

Piston motion for wave generation in this study uses three amplitudes and two wave periods for three water depths as presented in Table 1. Table 2 shows ratio of water depth and wave length as a guide to judge whether tested water depths are deep or shallow and their degree. Here wave length, λ, is calculated by linear wave theory given as Eq.1. Input gain for the wave maker has influence to piston movement and increase of input gain linearly increases wave amplitude. In order to realize the same amplitude of piston motion for different water depths and wave periods, the input gain for wave maker is carefully adjusted by trial and error.

Table 1 Period and amplitude of input piston motion

Period: T (s)	
1.15	1.95
Amplitude: A (mm)	
16.5	16.5
33.0	33.0
50.5	50.5

Table 2 Water depth and h/λ ratio

Water depth: h [mm]	T=1.15 [s] h/λ	T=1.95 [s] h/λ
1100	0.53	0.21
750	0.37	0.16
400	0.22	0.11

$$\lambda = \frac{gT^2}{2\pi}\tanh\left(\frac{2\pi h}{\lambda}\right) \quad (1)$$

3. NUMERICAL SIMULATION

3.1 Smoothed Particle Hydrodynamics

SPH is a fully Lagrangian mesh-less method. The technique discretises a continuum using moving particles (evaluation points) and physical quantities (position, velocity, density and pressure) are computed as interpolation values of neighbouring particles. SPH was firstly derived for astrophysical field and is widely used to solve free surface flow problems[3] represented by dam breaking. SPH based on mathematical fundamentals on integral, any function $F(r)$ can be computed by integral approximation (Eq.2) and to calculate contribution of the neighbor particles using kernel function. In this study, we use Wendland quintic function given by Eq.3. Momentum conservation equation in SPH is Eq.4 and for effect of dissipation becomes as Eq.5. The equation of viscosity term (Eq.6) was introduced by Monaghan[4]. Fluid in SPH is treated as weakly compressible and equation of state is used to determine fluid pressure using Eq.7. Equation of delta SPH is introduced for a diffusive term to reduce density fluctuations, as the result continuity equation becomes to Eq.8 where the Delta-SPH δ_ϕ coefficient of 0.1 is used in our simulation.

$$F(r) = \int F(r')W(r - r', h)dr' \quad (2)$$

$$W(r,h) = \alpha_D\left(1 - \frac{q}{2}\right)^4 (2q+1) \quad 0 \le q \le 2 \quad (3)$$

$$\frac{dv}{dt} = -\frac{1}{\rho}\nabla P + g + \Gamma \quad (4)$$

$$\frac{dv_a}{dt} = -\sum_b m_b\left(\frac{P_b + P_a}{\rho_b \cdot \rho_a} + \Pi_{ab}\right)\nabla_a W_{ab} + g \quad (5)$$

$$\Pi_{ij} = \begin{cases} \frac{-\alpha c_{ab}\mu_{ab}}{\rho_{ab}} & V_{ab} \cdot r_{ab} < 0 \\ 0 & V_{ab} \cdot r_{ab} > 0 \end{cases} \quad (6)$$

$$P = \frac{c_0^2 \rho_0}{7}\left[\left(\frac{\rho}{\rho_0}\right)^7 - 1\right] \quad (7)$$

$$\frac{d\rho_a}{dt} = \sum_b m_b v_{ab} \cdot \nabla_a W_{ab} + 2\delta_\phi h c_0 \sum_b (\rho_b - \rho_a)\frac{r_{ab} \cdot \nabla_a W_{ab}}{r_{ab}^2}\frac{m_b}{\rho_b}) \quad (8)$$

In this study, an open source SPH solver of Dualsphysics ver.4.0 is used. Development of DualSPHysics started from SPH formulation implemented in SPHysics but not optimized for large-scale simulations. Dualsphysics is implemented in CPUs using openMP or on GPU for parallelization to maximize calculation speed during computation[5]. Dualsphysics can be downloaded at www.dual.sphysics.org.

3.2 Calculation condition

Schematic view of a numerical wave basin is given in Fig.4. The dimension of wave basin, position of wave gauge and pressure sensor exactly correspond to the experiment. Calculation condition that used in DualSPHysics is described in Table 3. We try to reproduce wave generation and propagation in the wave basin by imposing the measured piston motion to numerical simulation as the moving wall. Parallel computing in DualSPHysics can be executed on GPU, so significant reduction of computation time can be achieved. Wave surface is detected at the WG position and pressure is calculated at the pressure sensor position in the simulation.

All simulations are performed in 2-D because only single-directional waves are generated in the experiment. In this paper, we only run numerical simulation for the wave period of 1.95 seconds to focus shallow water effects in which existence of seabed is influential certainly. Initial particle distance and total number of particles are given in Table 4. Constant particle distance of 3 mm is used for all the water depths and is determined so as to be included over 15 particles in the vertical direction for the minimum wave height among selected cases. Relatively large number of particles (spatial resolution) is used for 2-D simulation to discuss the validity of SPH for the wave generation and propagation.

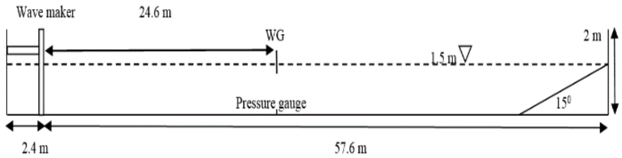

Fig.4 Schematic view of numerical wave basin

Table 3 Calculation condition

Kernel function	Wendland
Time step algorithm	Sympletic
Viscosity treatment	Artificial with α=0.01
Coefsound	20.0
Particle distance [mm]	3.0
Coefh	1.5
CFL number	0.3
Delta-SPH	0.1
Duration of simulation [s]	40.0

Table 4 Particle distance and total number of particles

Initial Particle distance [mm]	Water depth h [mm]	Total number of particle
3.0	1100	6,724,154
	750	4,521,922
	400	2,388,834

4. RESULTS AND DISCUSSION

Time histories of the piston motion with amplitudes of 33.0 and 50.5 mm are presented in Fig.5. These time records are used as the input data for the piston movement in the SPH simulation. Fig.6-7 show comparisons of water elevation between the experiment and the SPH simulation at the 24.6 m distance from the neutral position of the piston. The piston motion starts at t=0 sec both in the experiment and the simulation. Table 5 shows the wave amplitude in steady state obtained from the experimental and numerical results as well as a linear wave maker theory known as Biesel transfer function given in Eq.9.

In the experiment, the wave amplitude decreases with water depth and phase velocity also decreases. This is well known features of seabed effects and the SPH simulation well explains the physical trend of shallow water waves. In both conditions of A=33.0 and 50.5 mm, SPH result shows good agreement in steady wave amplitude, developing rate of wave amplitude and phase velocity, i.e. reaching timing, for the water depths of 1100 and 750 mm. The maximum error of these conditions is less than 3.0 % even the measurement point is far from the wave maker. In case water depth is 400 mm, the accuracy of reproduction of wave amplitude and phase velocity becomes worse slightly. This could be because the number of particles in vertical direction becomes smaller when the wave amplitude becomes smaller due to seabed effects consequently. This guess is supported by a grid study, using 3 and 6mm as the initial particle distance presented in Fig.8, because the numerical result using smaller number of particles results in larger wave crest.

All the SPH simulations are run on GeForce GTX TITAN X 12GB GDDR5. The total number of particles for water depth of 1.1 m is the highest, 6.7M, among three cases and computation time is about 51.5 hours for 40 seconds simulation.

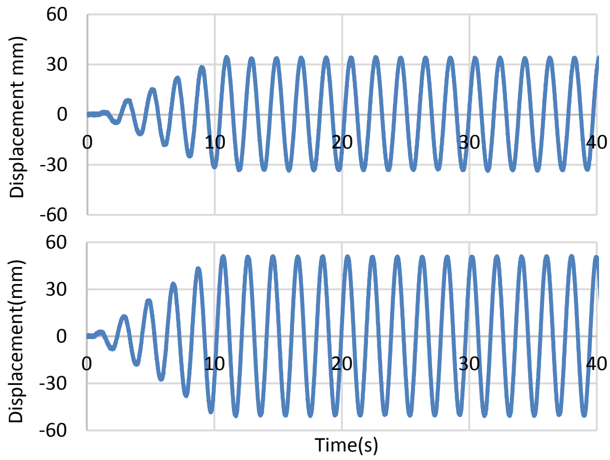

Fig.5 Time history of piston motion with A=33.0 and 50.5 mm

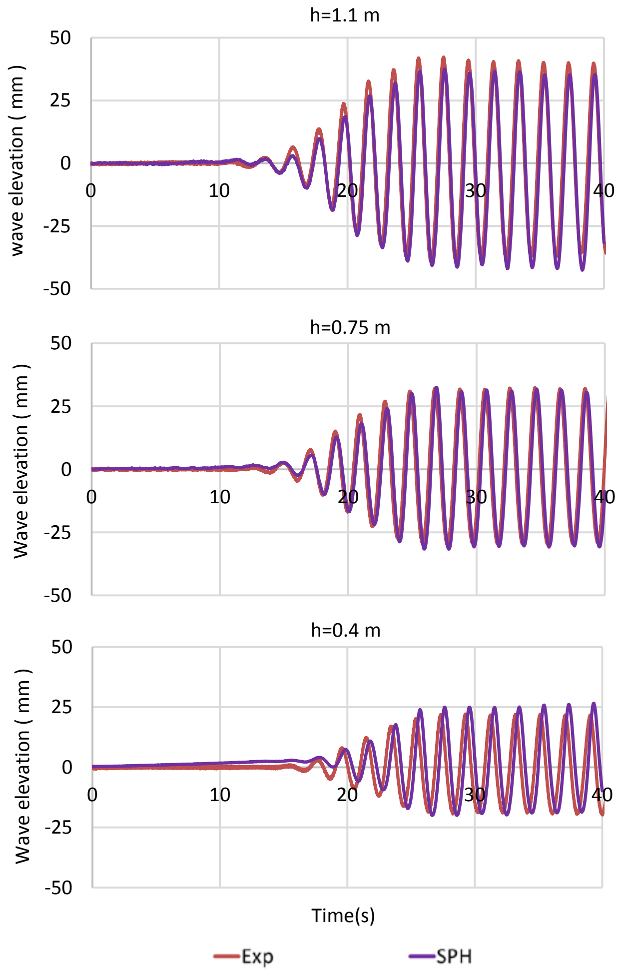

Fig.6 Comparison of water elevation between experiment and simulation with A=33.0 mm

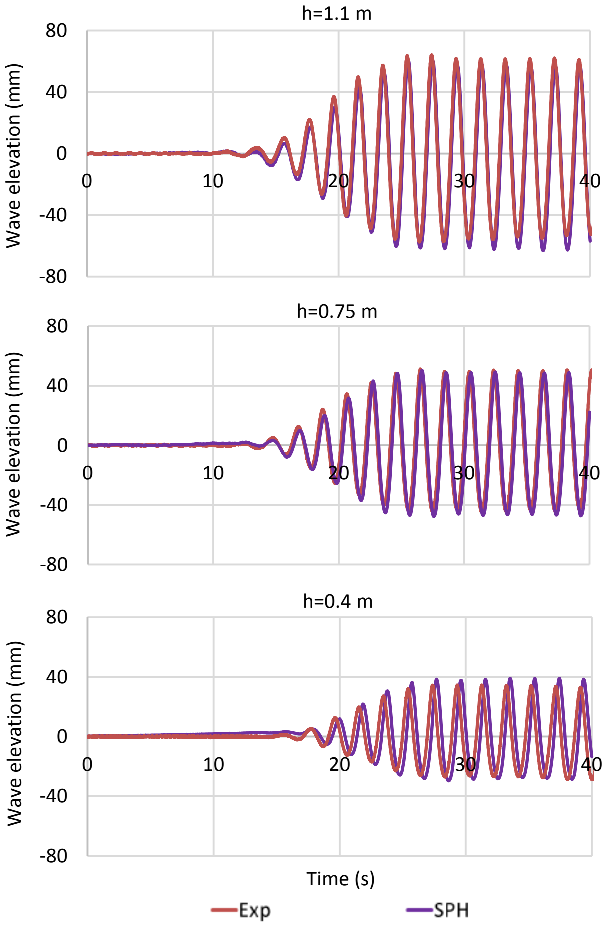

Fig.7 Comparison of water elevation between experiment and simulation with A=50.5 mm

Table 5 Steady wave amplitude

h	A	Exp.	SPH	Theory
1.1	33.0	38.9	38.5	41.9
	50.5	57.9	59.3	64.2
0.75	33.0	31.0	30.1	33.2
	50.5	46.3	47.3	50.8
0.4	33.0	19.6	21.8	23.0
	50.5	29.8	32.7	35.2

$$\zeta_a = \frac{2\sinh^2(\frac{2\pi h}{\lambda})}{\frac{2\pi h}{\lambda}+\sinh(\frac{2\pi h}{\lambda})\cosh(\frac{2\pi h}{\lambda})} \quad (9)$$

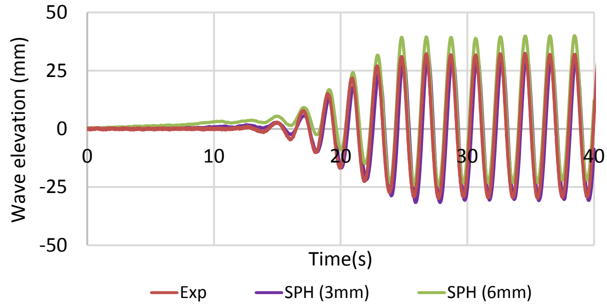

Fig.8 Grid study for water depth of 750 mm with A=33.0 mm

Fig.9 shows an example of numerical result of wave propagation along the basin and Fig.10-11 do visual comparison of wave profile for water depth of 1100 and 400 mm. The center of video image corresponds to the position of wave gauge. Fig.9 clearly demonstrates the SPH can reproduce wave propagation with less energy dissipation. Figs.10-11 show that the SPH result of wave profile looks very similar with the experimental result for the both water depths.


Fig.9 Numerical result of wave propagation for h=1100 mm

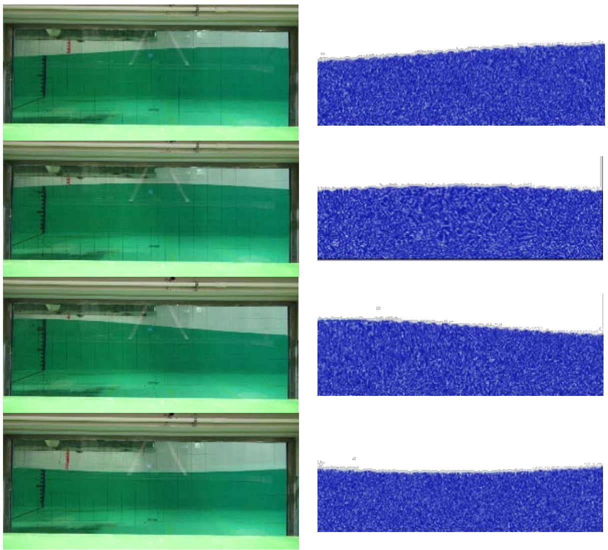
Fig.10 Comparison of wave profile for h=1100 mm with A=50.5 mm

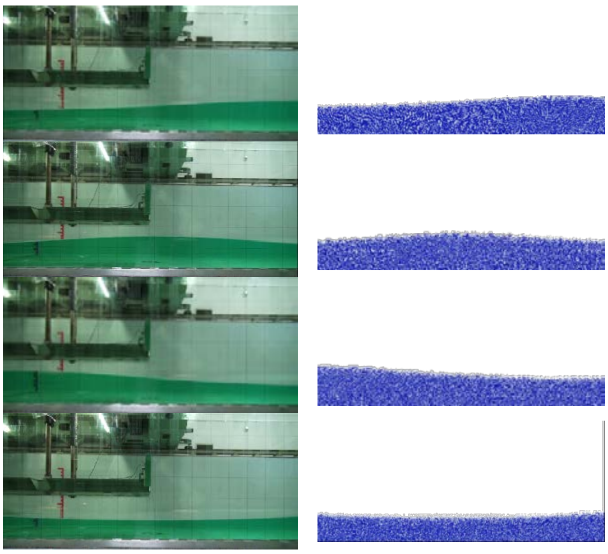
Fig.11 Comparison of wave profile for h=400 mm with A=50.5 mm

Prior to discussion on wave pressure, verification of hydrostatic pressure calculated by SPH is done. The hydrostatic pressure at the depth of pressure sensor is calculated without the movement of piston and is shown in Table 6. The error of the SPH result is less than 2% and smooth pressure gradient is obtained as shown in Fig.11.

Table 6 Hydrostatic pressure

Analytical solution	SPH 5-second average	Error
10.7 (kPa)	10.5 (kPa)	1.87 (%)

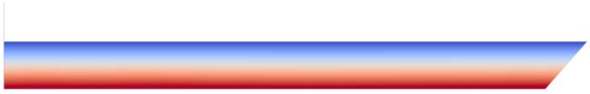

Fig.11 Contour of hydrostatic pressure for h=1100 mm

Fig.12 shows calculated pressure obtained at different depth. Here depth of 1100 mm means that the pressure sensor is located at the bottom of numerical basin. Unfavorable fluctuation with wide frequency and large amplitude appears for the depth of 1100 and 600 mm, whereas it does not so much for the depth of 100 mm. Since this pressure fluctuation is obviously unphysical, the pressure at relatively deep depth is unusable for engineering problems. Fig.13 shows a comparison of wave pressure between the experiment and SPH. Here the numerical result is obtained at 0 mm depth, which corresponds to the surface of calm water, so negative pressure cannot be detected in the numerical result. Although the numerical fluctuation still exists to some extent, the representation of wave pressure is acceptable from a practical view point.

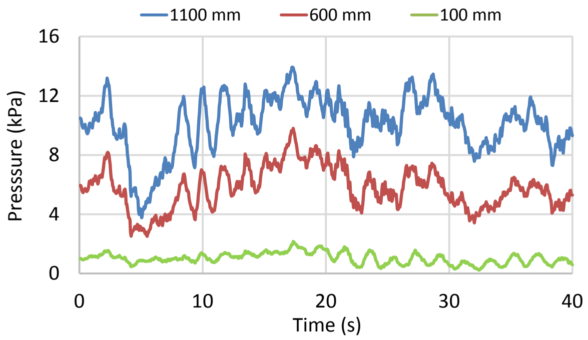

Fig.12 Calculated pressure at different depth with h=1100 mm and A=50.5 mm

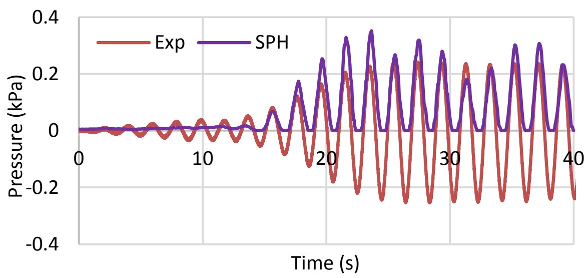

Fig.13 Comparison of wave pressure between experiment and simulation with h=1100mm and A=50.5 mm

5. CONCLUSIONS

The validity of water waves reproduced by the SPH method is investigated through comparisons with a sophisticated experiment. SPH simulation, which directly uses the measured motion of piston-type wave maker, shows promising results in the reproduction of wave generation and propagation in shallow waters. Wave pressure is also reproduced in practically acceptable level. The similar investigation for shorter waves, i.e. more steep waves, is necessary.

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