Study of Non-Newtonian Fluids by DualSPHysics

Montréal, October 7 2020

3rd follow up meeting

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Outline

- Newtonian Fluid by HBP
- Maximum Velocity and Boundary Velocity
- Poiseuille Flow of Bingham Fluid
- Bingham Fluid by HBP

Simulation Set up for Poiseuille Flow

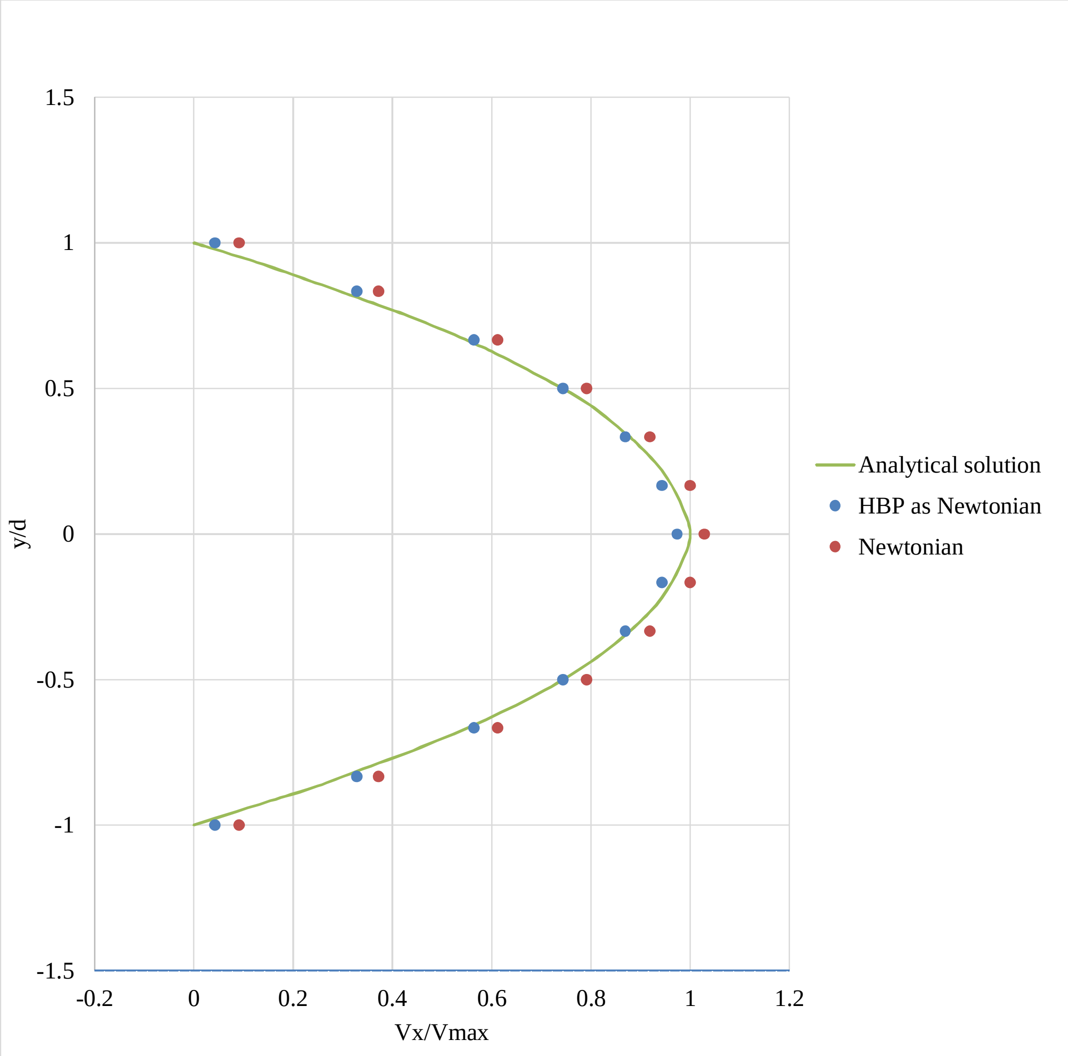
Poiseuille flow characteristics:

- Distance between plates is 1 mm.
- The density of fluid is 1000 kg/m^3.
- The kinematic viscosity is 1e-6.
- The acceleration is 1x10^{-4} m/s^2.
- Physical time is 1 s.

Simulation set up	Inter-particle distance (IDP)	IDP to thickness ratio	Physical time (s)	Kinematic viscosity (m2/s)	Run time (h)
Constant viscosity	0.02 mm	0.02	1	1 e −6	1
HBP as Newtonian	0.02 mm	0.02	1	1 e −6	34

By using HBP model as Newtonian fluid run time for IDP/Thickness=0.01 would be 8 days..!

Velocity Profile of Poiseuille Flow

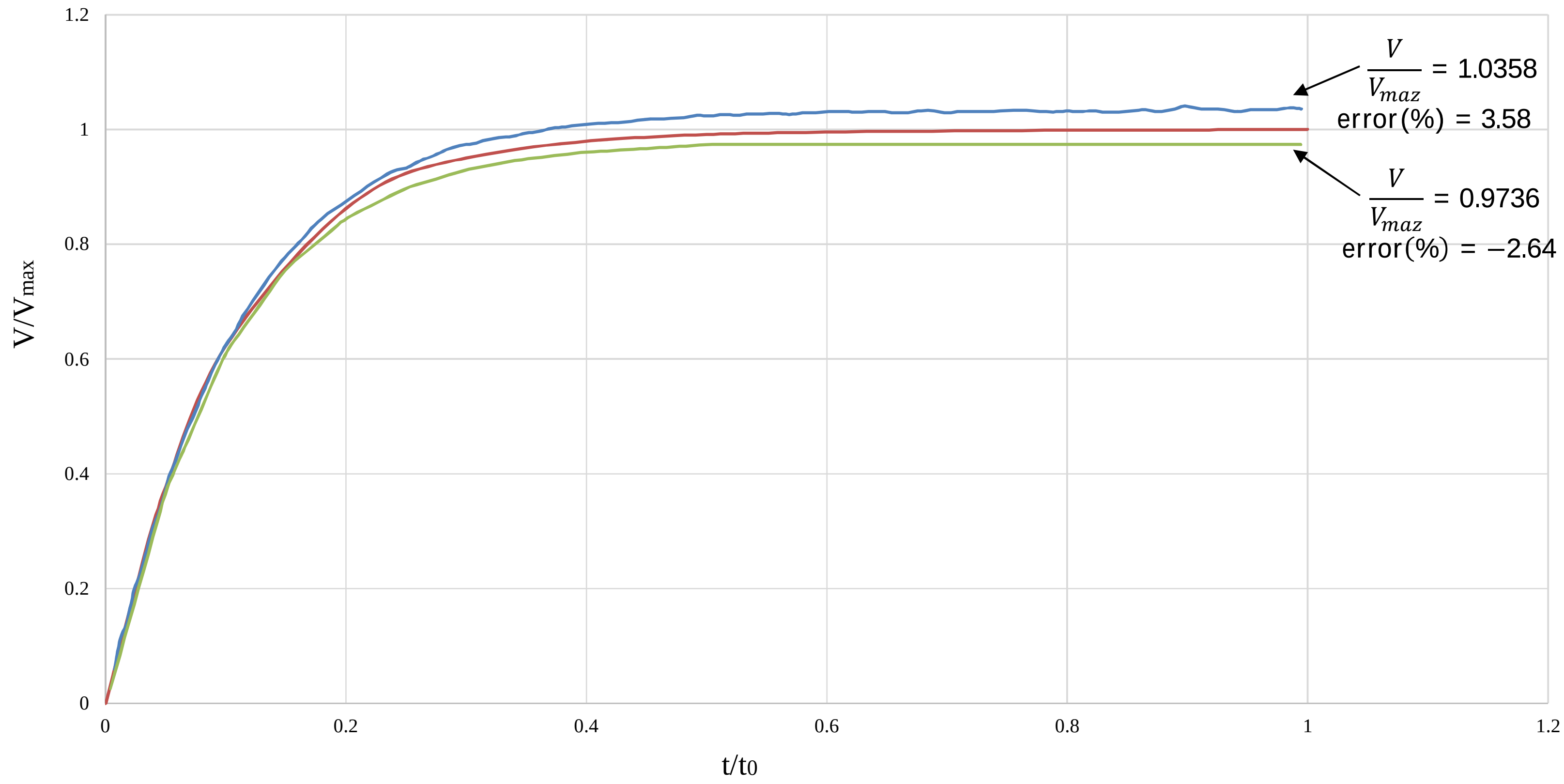


$$\frac{IDP}{Thickness} = 0.02$$

$$HBP\ as\ Newtonian\ run\ time = 34\ h$$

$$Newtonian\ run\ time = 1\ h$$

Maximum Velocity at the Mid-plane



$$V_{max} = \frac{H^2 \Delta P}{8 \mu L}$$

$\frac{\Delta P}{L}$ = pressure gradient

H = distance between plates

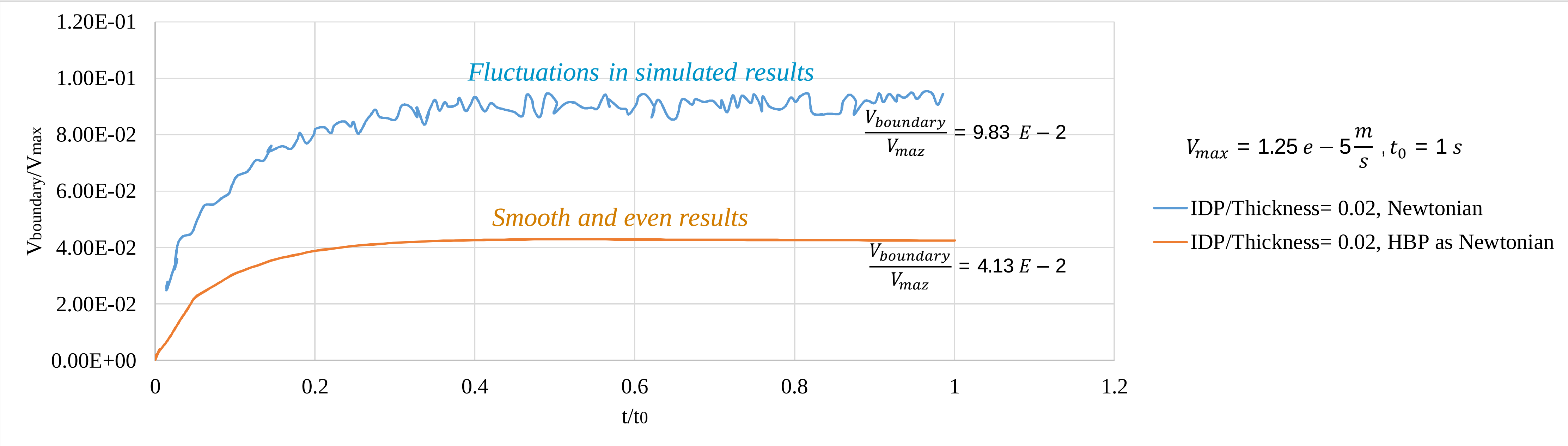
μ = viscosity

$$V_{max} = 1.25\,e-5 \frac{m}{s},\ t_0 = 1\ s$$

- Analytical solution, Delta P/L 0.1
- IDP/Thickness= 0.02 Newtonian
- IDP/Thickness= 0.02 HBP as Newtonian

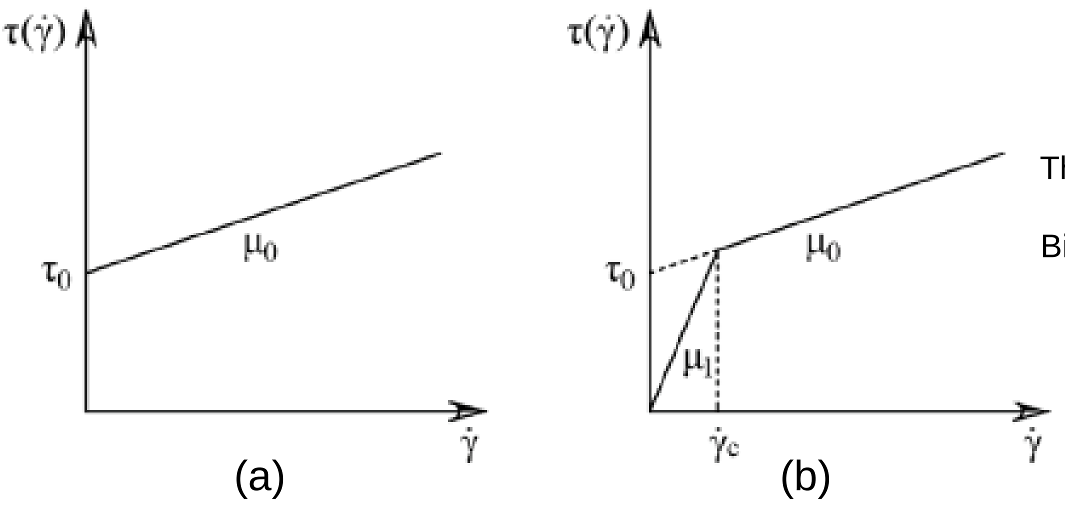
Comparable with IDP/Thickness=0.008 using constant viscosity

Boundary Velocity



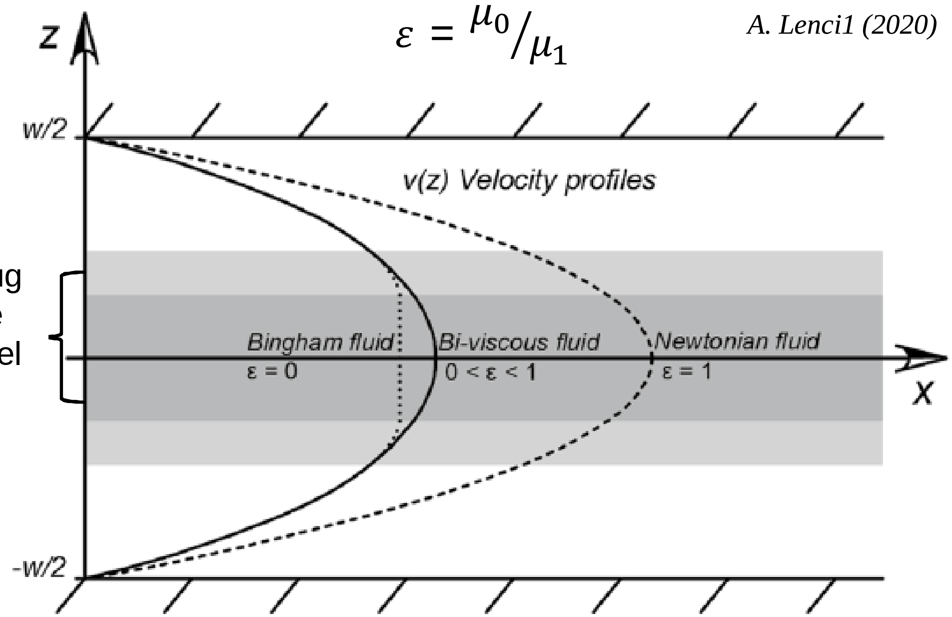
Poiseuille Flow of Bingham Fluid

- Yield stress fluids exhibit a threshold stress that separates different fluid behaviours.
- Bingham model is the simplest yield model.
- The flow in the yielded domain is described by Navier-Stokes equations.
- The plug region where the yield stress is not exceeded.



(a) Bingham model and (b) bi-viscosity model

The typical plug region of the Bingham model is shaded in darker grey.



Parallel plate scheme with velocity profiles for a Bingham fluid, bi-viscous fluid, and Newtonian fluid

Velocity Profile of Bingham Fluid

The flow of a Bingham fluid between two plates separated by a distance 2H, taking into account the steady-state conditions, constant cross section, absence of gravitational effects and isothermal flow, and being incompressible,

$$\rho\left[\frac{\partial \vec{v}}{\partial t}+\vec{v}\cdot\nabla\vec{v}\right]=\nabla\cdot\overline{\overline{\tau}}-\nabla P+\rho\vec{g}$$

The pressure gradient effect is considered as a favourable driving force for fluid movement.

L. V. Ortega (2019)

$$\nabla P \cong \frac{\Delta P}{\Delta z}=\frac{(P_L-P_0)}{L}=\frac{\Delta P}{L} \quad \text{and} \quad \frac{d\tau_{yx}}{dy}=\left(\frac{\Delta P}{L}\right)$$

Substituting Bingham model in shear stress, we obtain:

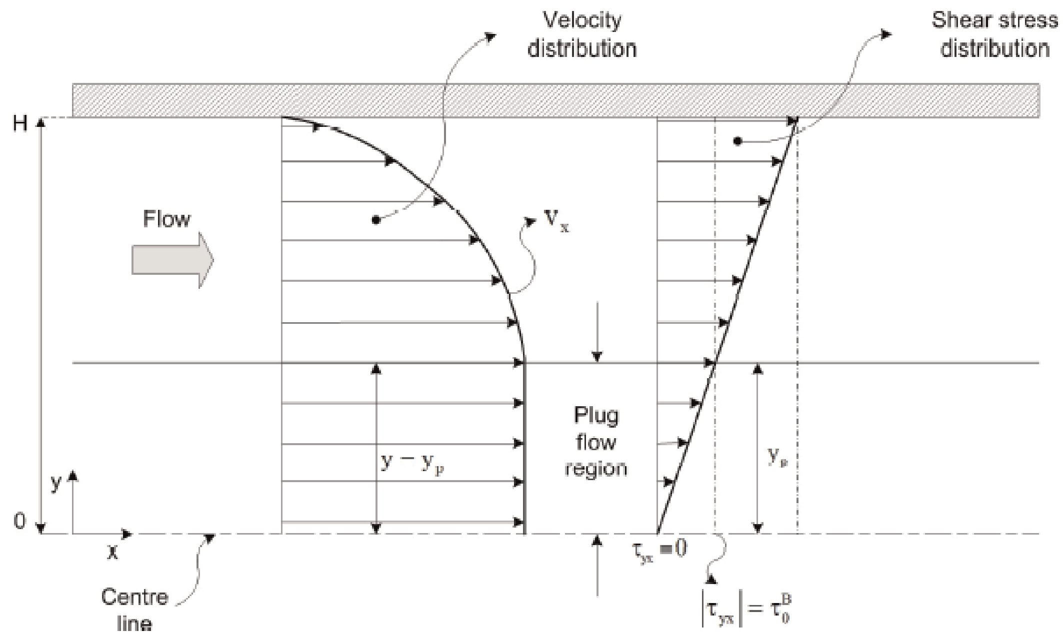
$$\tau_{yx}=\left(\frac{\Delta P}{L}\right)y \quad \text{and} \quad \overbrace{\mu_B\left(\frac{dv_x}{dy}\right)+\tau_0}^{\text{Bingham model}}=\left(\frac{\Delta P}{L}\right)y$$

Performing the corresponding integrals and applying boundary condition, we have:

$$v_x=\frac{\tau_0}{\mu_B}H\left(1-\frac{y}{H}\right)+\left(\frac{\Delta P}{L}\right)\frac{H^2}{2\mu_B}\left[\left(\frac{y}{H}\right)^2-1\right]$$

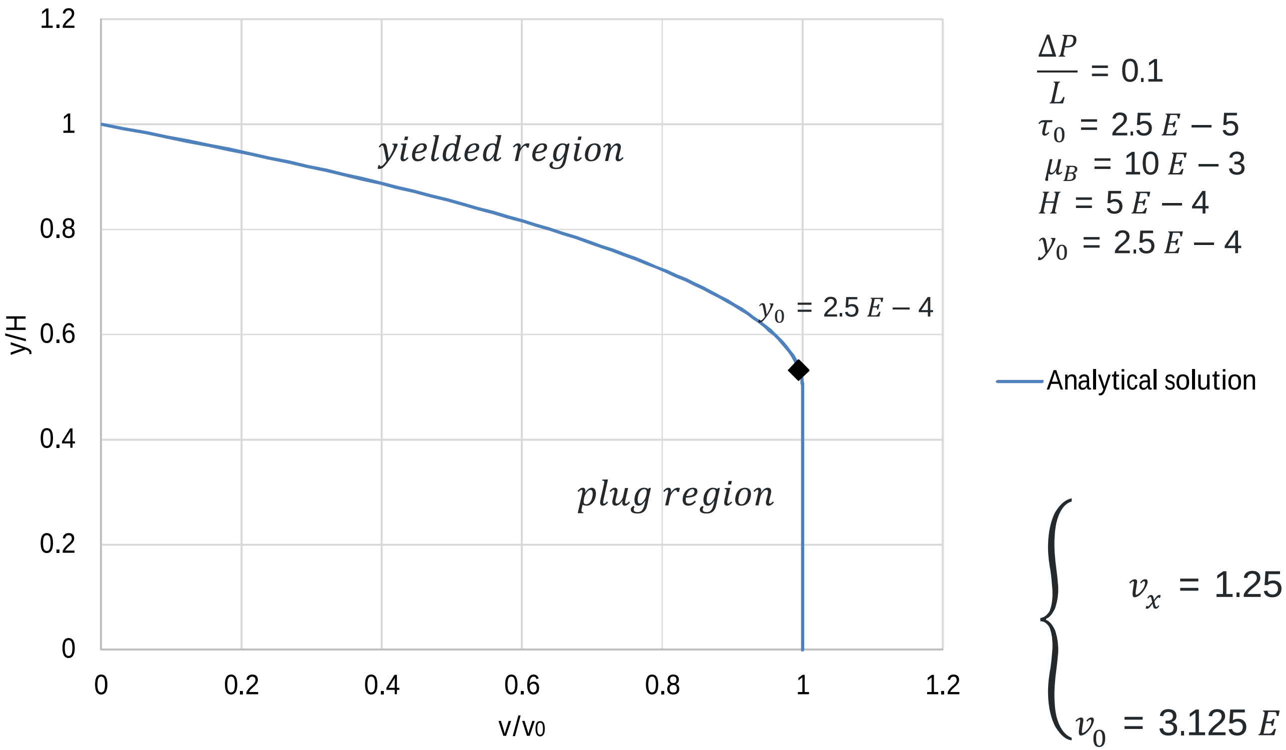
For velocity profile in the plug region, the condition of the yield stress is proposed

$$y=y_0;\quad \tau_0=\left(\frac{\Delta P}{L}\right)y_0 \qquad v_0=\left(\frac{\Delta P}{L}\right)\frac{H^2}{2\mu_B}\left(1-\frac{y_0}{H}\right)^2$$



Flow of a Bingham fluid between two plates one half view

Analytical Solution for Bingham Fluid



$\frac{\Delta P}{L} = 0.1$
$\tau_0 = 2.5\,E-5$
$\mu_B = 10\,E-3$
$H = 5\,E-4$
$y_0 = 2.5\,E-4$

$$\begin{cases} \text{velocity profile in the yielded region} \\ v_x = 1.25\,E-5 \times \left(\left(\frac{y}{5\,E-4}\right)^2 - \left(\frac{y}{5\,E-4}\right)\right) \qquad y_0 \le |y| \le H \\ \text{velocity profile in the plug region} \\ v_0 = 3.125\,E-6 \qquad 0 \le |y| \le y_0 \end{cases}$$

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Bingham Fluid by HBP Model

- The Bingham model exhibits a discontinuity when the shear rate is zero and *viscosity goes to infinity*.
- The Herschel-Buckley-Papanastasiou has a finite value viscosity over the yielded region.

$$\mu_{eff} = K(\dot{\gamma})^{n-1} + \frac{\tau_y}{2\dot{\gamma}}(1 - e^{-2m\dot{\gamma}})$$

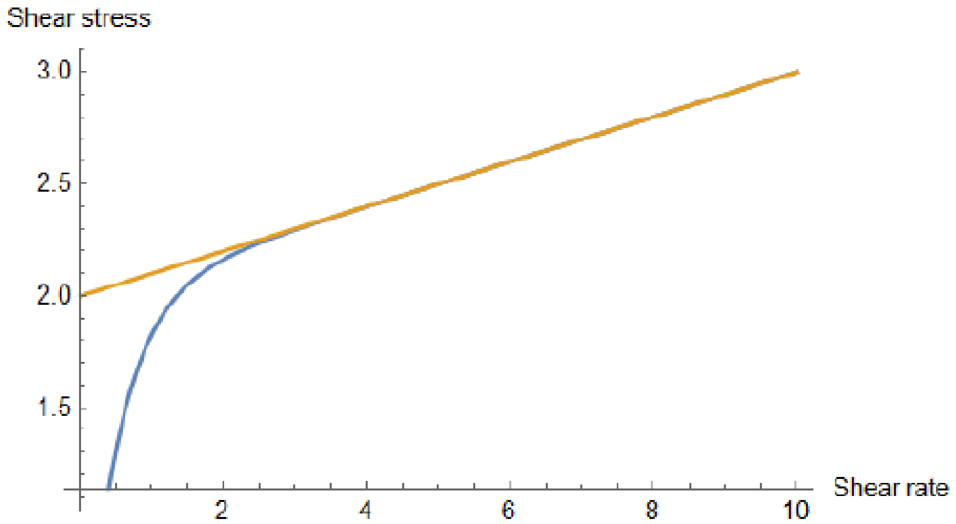
τ_y *is the yield stress of the fluid*

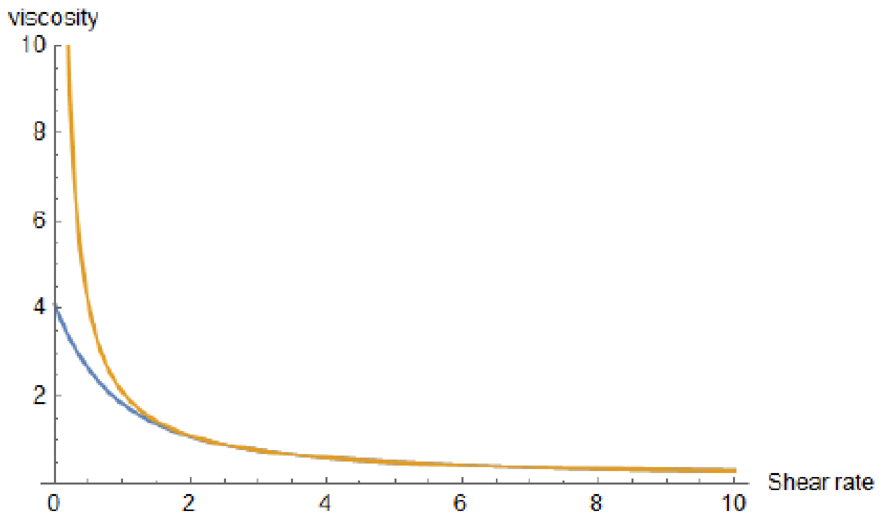
k *is flow consistency index*

n *is the flow behavior index*

m *is controlling factor*

DualSPHyisics documentation (2020)





HBP model does not exhibit a discontinuity at zero shear rates in contrast with a pure Bingham model.

Bingham Fluid Parameters in HBP Model

The following parameters are defined in HBP model to simulate Bingham fluid.

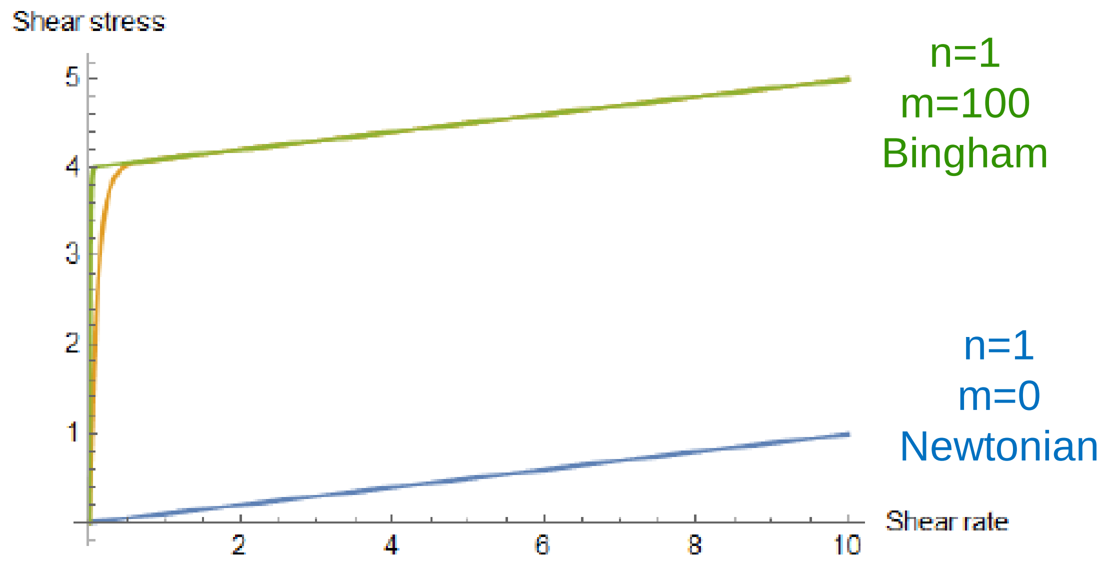
Kinematic viscosity
Visco= 1E-6

Maximum specific yield stress
tau_yield= 2.5E-5

Papanastasiou exponential stress growth
HBP_m= 100

Flow behavior index
HBP_n= 1

DualSPHyisics documentation (2020)



Bingham model fluid parameters